TMSP 2021/22
Problems 4 (28.03.2022)

1 Tricritical point.

The tricritical point is defined as the point in the phase diagram at which a line of first-order transitions turns into a line of critical points, i.e., upon crossing this point (along the line) first-order transitions go over into critical points.
Consider the Landau theory for a uniform uniaxial ferromagnet with the following form of the effective Hamiltonian

$$\mathcal{H}_{eff}(m) = G_0(T) + V\left(\frac{a\tau}{2}m^2 + \frac{u}{4!}m^4 + \frac{v}{6!}m^6 - h\,m\right),$$

where $a, v > 0$, while the parameter u can have arbitrary sign. For $h = 0$, fixed values of $a = a^*$, $v = v^*$ plot the phase diagram in variables (u, τ), i.e., identify the phases, the order of the phase transition, and find the corresponding tricritical point (u_{tr}, τ_{tr}), i.e., determine the values u_{tr} and τ_{tr}. Determine the value of the critical index γ_{tr} upon approaching the tricritical point along the direction $u = u_{tr}$, i.e., $\chi_T \sim |\tau - \tau_{tr}|^{-\gamma_{tr}}$.

2. Using the Landau theory determine the magnetization-magnetization correlation function $G_{mm}(x', x'', T, h = 0)$ for $d = 1$ $(V = \infty)$.

3. For the one-dimensional Ising model with periodic b.c. check by direct calculation if the equality $\chi_T(T, h = 0) = \beta \sum_{n=-\infty}^{\infty} \Gamma(n, T, h = 0)$ is true.
Check if analogous expression
$\chi_T(T, h = 0) = \beta \int d\vec{r}\, G_{mm}(\vec{r}, T, h = 0) = \tilde{G}_{mm}(\vec{q} = 0, T, h = 0)$
also holds in Landau theory $(V = \infty)$.